# A Hyperbolic Behaviour Model for Partially Saturated Fine Soils

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Abstract: - Constitutive models of partially saturated soils have been widely used in geotechnical engineering. Most of these models depend on complicated parameters; among them, soil suction is the most important one. However, the equipment and level of expertise required for determination of this parameter during a test procedure is beyond the capabilities of most of geotechnical laboratories. Thus, a behaviour model independent of soil suction may be very interested instead of such complicated laboratory measurements.
In this research, a new behaviour model of partly saturated soils is presented. This model is constructed based on a series of collected data from different authors and validated with their experimental results. Employing a dramatic and continuous transition of soil strengths from a fully saturated to fully dry condition is the base of this model. Soil parameters which were used in this approach are moisture content, plasticity indices, saturated-drained internal friction angle and unconfined compressive strength of soil in dry and hardened states. Based on presented equation in this research and introduced effective parameters, it is possible to determine partially saturated soils shear strengths without any measurement of soil suction.

Key-Words: Partially saturated soils, Constitutive models, Failure criterion, Hyperbolic function, Matric Suction

## 1 Introduction

Conventional theories that describe soil geomechanical behaviour are often assumed a saturated state for soils. This hypothesis is true in most of natural conditions, however, soil partially saturated behaviour is greatly different of its saturated state, especially in fine grained soils that show more different behaviour in saturated and partially saturated conditions. Most of ground layers are partially saturated in Iran, especially in the middle parts of the country and locations with deep ground water table. The problems involving in laboratory tests and sophisticated theories in partially saturated soils, such as direct soil suction measurement and many affecting factors, cause the development and understanding of partially saturated soil behaviour to be hard and very slow [1]. Many relationships, theoretical procedures and behaviour models of partially saturated soils have been developed in the recent years and presented in the last two decades. By the other hand, stability and stiffness of cohesive soils are highly dependent on their moisture contents. Decrease in moisture content of them causes a softness reduction as far as they act similar to soft-rocks. Thus, a failure criterion of soft-rocks is applicable for partially saturated soils in low ranges of moisture contents.

A behaviour model is a mathematical representation of the response of a continuous medium. Partially saturated soils behaviour models are different of those for saturated soils. Consideration of surface tension as well as soil suction is very significant in unsaturated soils [2]. Effective stress relationships in partially saturated soils are often determined based on parameters which depend on soil degree of saturation. Matric suction is also considered as another important variable in unsaturated soils, when a stress state is demonstrated. Thus, control and measurement of matric suction to evaluate physical properties (fluid flowing, strength and volume change) of unsaturated soils under different stress states are of great importance. Many problems are involved in negative pore water pressure measurement causing a scientific limitation [3]. Many types of behaviour models have been presented for partially saturated soils. Based on these models, the failure criterion is reached in three possible cases: variations of net mean stress, variations of deviator stress and variations of matric suction. Partially saturated soil behaviour models are divided into two major classes: Elastic models and elasto-plastic models. Elastic models exhibit the relationships between strain increase with net total stress and matric suction. Among them, Fredlund and Morgenstren (1976) and Lloret et al. (1987) models can be pointed here. Wheeler and Karobe (1996), represented reviewed the suggested behaviour models in a comparative report.

Elasto-plastic models are divided into two subgroups: expansive models and non-expansive models. Barcelona constitutive model is among the first models in partly saturated soils, based on a presented theory of Alonso and his co-workers (Alonso et al., 1990). This model is a generalized Modified Cam-Clay (MCC) model in fully saturated soils for partially saturated soils in which, the concept of loading-collapse was employed. This model makes it possible to represent a range of important partially saturated soils behaviours, i.e. collapse in wetting process in a steady state soil mechanics representation for partially saturated soils, and thus can be considered as a basic concept in development of many elasto-plastic models [4].

Most of valid behaviour models in partly saturated soils are elasto-plastic models. Alonso et al. (1990), Wheeler and Sivakumar (1995), Bolzon et al. (1996) and Thomas and Hi (1998) are among the well known models. This is mentionable that many of investigators, who are experienced and active in numerical analysis, have interested in Thomas and Hi model [5-9].

In this study, a new model is presented to demonstrate the partially saturated behaviour of fine clayey soils based on a set of a reliable data base and experimental results presented by other researchers. This model is based on a gradual and continuous transition of soils from fully saturated to fully dry states. Soil water content, plasticity indices, saturated-drained internal friction angle and dry-hardened unconfined compressive strength are the model parameters.

A critical state theory reflects the behaviour of fully saturated soils. It is possible to extract shear strength and critical state parameters from laboratory triaxial tests for saturated soils. In fully dry state, the clays are lumped together into a clod and behave like soft rocks as far as they transformed into clay-stone under higher confining stresses. Thus, evaluation of clays in dry state can be performed in the light of soft rock conventional theories. This is almost valid for clayey silts. Conventional shear strength criteria of rocks (like Hoek and Brown, 2000) are often a nonlinear relationship that has an intercept (latitudinal distance of the origin) known as cohesion and an asymptote as the extreme limit of shear strength. In the other words, isotropic stresses can affect shear strength of rocks as far as they show a compressible soil-like behaviour.

Consequently, transition of clays from fully saturated to fully dry states and vice versa, must satisfy the abovementioned conditions. In the other words, the intermediate behaviour must be something between these two extreme states, i.e. fully saturated and fully dry, while the strength transition from fully saturated (Mohr-Coulomb criterion) to fully dry condition (Hoek-Brown criterion) should be continuous. Figure 1 shows these conditions.

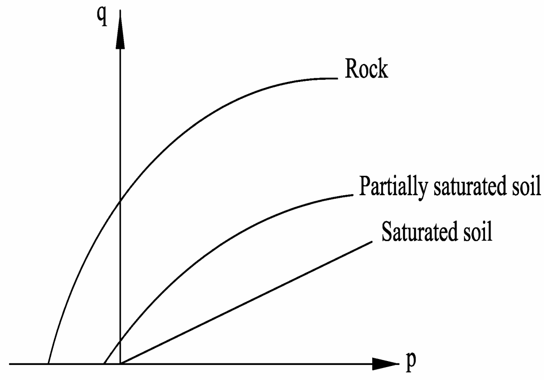


**Fig. 1.** Mechanical behaviour transition of soils in saturated to fully dry states

According to figure 1, the function that can describe the real behaviour of partly saturated soils is an increasing curve with an intercept considerably smaller than rock behaviour equations and with an intermediate slope between dry and fully saturated states. By the other hand, variations of the parameters of this function must accurately transform this function into the two extreme cases, i.e. fully saturated and fully dry states. Hyperbolic tangent function is among the applicable functions to display such a behaviour. The general form of a hyperbolic tangent function is as follow:

$$f(x) = a\tanh(b(x+c)) \tag{1}$$

## 2 Data Base

The behaviour model for partially saturated soils is presented based on the available information and experimental data that has been acquired among many published papers by well known researchers in partially saturated soils field.

According to required data for the model, the references have been chosen in a way that they can satisfy the following conditions:

Soil type is clay or a silt-clay mixture.

Soil index and physical properties, i.e. moisture content and Atterberg limits are mentioned.

Confining stress and shear stress of soil versus degree of saturation or moisture content are specified.

The references that have been used in this research are Alonso et al. (1990) [5], Wheeler and Sivakumar (1995) [7], Blatz et al. (2002) [10], Adams and Wulfsohn (1998) [11], Eko (1998) [12], Cetin (1999) [13], Rahardjo et al. (2004) [14], Montanez (2002) [15], Vanoudheusden et al. (2004) [16].
Since the modelling was performed in p-q space, the stresses are transformed into p-q space.
Figure 2 shows the final data base for modelling.

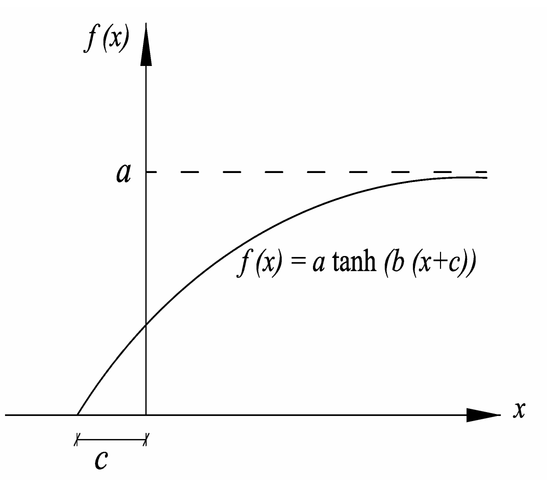


**Fig. 2.** Final data base for the model

It is mentionable that p and q are mean and deviator stresses defined as follow:

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \quad , \quad q = \sigma_1 - \sigma_3 \qquad (2)$$

Validation of model was performed based on experimental results on partially saturated fine grained soils [5].

## 3 Mathematical Model

Mohr-Coulomb model in drained condition along with Hoek-Brown criterion for rocks in p-q space are sequentially presented in equations 3 and 4 as follows:

$$\mathrm{M-C:} \quad \boldsymbol{q} = \boldsymbol{Mp} \qquad (3)$$

$$\mathrm{H\&B:} \quad \mathbf{q}^2 + \frac{m}{3} q_u \mathbf{q} - m q_u \mathbf{p} - q_u^2 = 0 \qquad (4)$$

Thus, these functions can be shown in the graphs of figure 3.

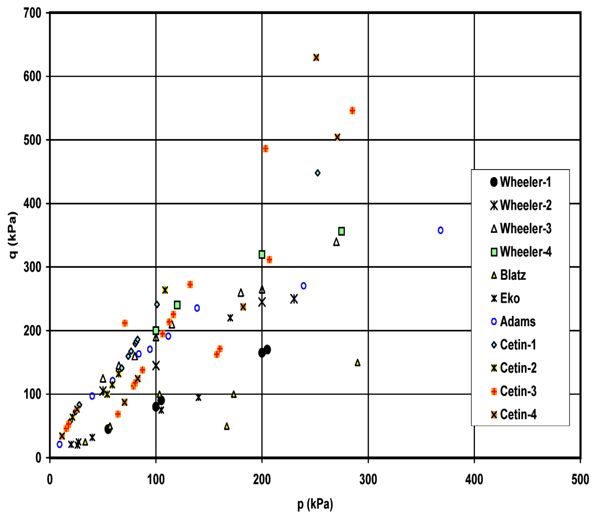


**Fig. 3.** Mohr-Coulomb and Hoek-Brown failure functions in (p,q) space

Figure 4, shows the parametric curve of equation 1 for a hyperbolic tangent function. As it is stated in this figure, parameters a, b and c in equation 1 stand for asymptote line, initial tangent and longitudinal distance from origin, respectively.

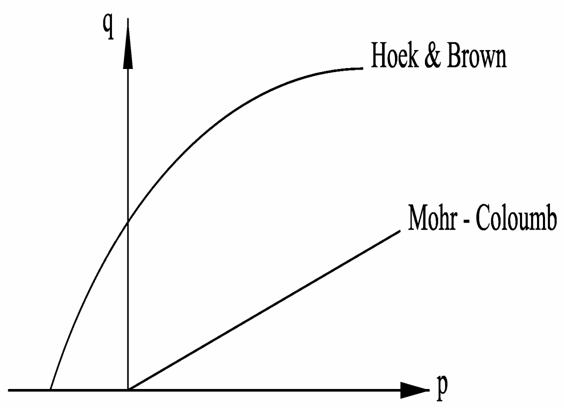


**Fig.4.** General form of a hyperbolic tangent function

According to figures 5 to 10, constant parameters of hyperbolic tangent functions are extracted from equation 1. These values are presented in table 1.

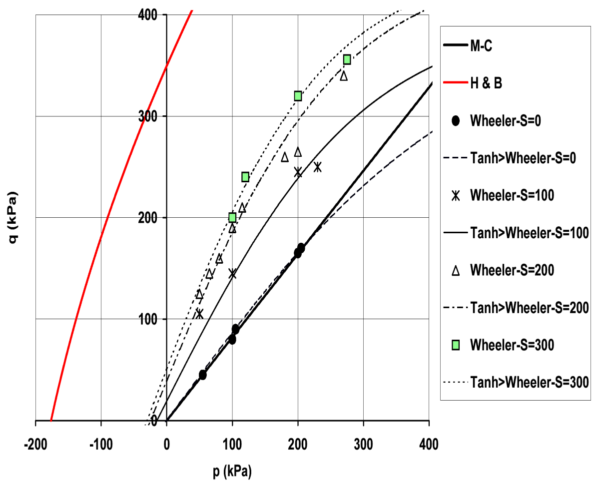


**Fig. 5.** Curve fitting for Wheeler and Sivakumar (1995) data by a hyperbolic tangent function

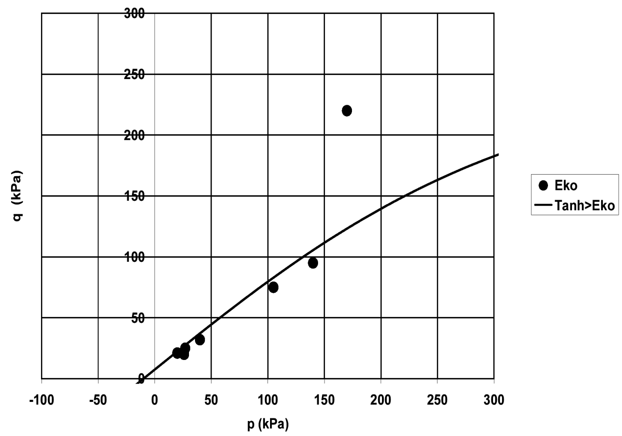


**Fig. 8.** Curve fitting for Eko (2005) data by a hyperbolic tangent function

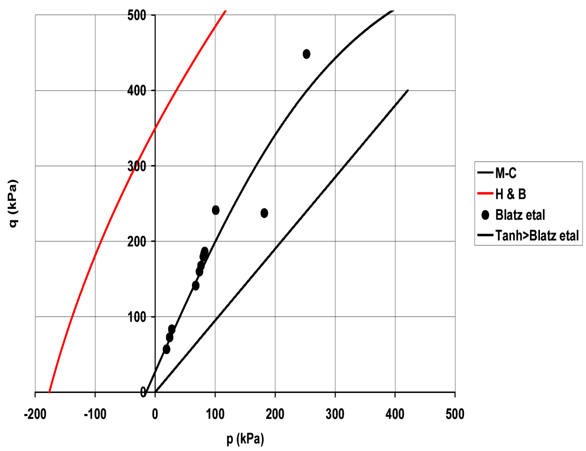


**Fig. 6.** Curve fitting for Blatz et al. (2002) data by a hyperbolic tangent function

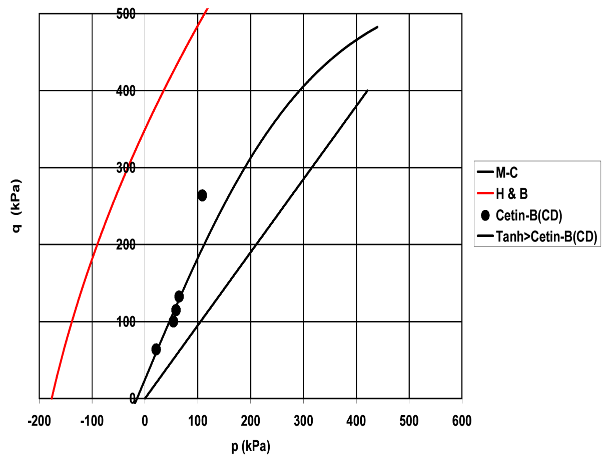


**Fig. 9.** Curve fitting of Cetin (1999) data for soil B as a hyperbolic tangent function

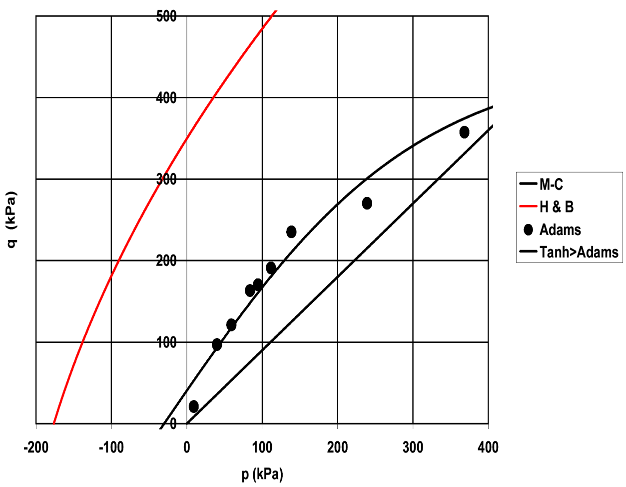


**Fig. 7.** Curve fitting for Adams and Wulfsohn (1998) data by a hyperbolic tangent function

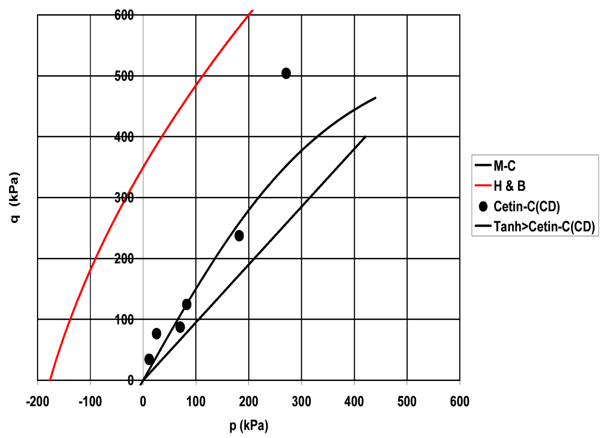


**Fig. 10.** Curve fitting of Cetin (1999) data for soil C as a hyperbolic tangent function

**Table 1.** Constants of hyperbolic tangent functions for different data

| No. | Reference | a | b | c |
|---|---|---|---|---|
| 1 | Wheeler & Sivakumar | 400 | 0.0032 | 15 |
| 2 | Blatz *et* al | 500 | 0.0026 | 10 |
| 3 | Adams & Wulfsohn | 450 | 0.0030 | 30 |
| 4 | Eko | 250 | 0.0030 | 10 |
| 5 | Cetin (Soil-B) | 550 | 0.0030 | 15 |
| 6 | Cetin (Soil-C) | 550 | 0.0028 | 0 |

According to obtained values for a, b and c parameters and based on effective parameters in determination of mechanical behaviour of partially saturated soils, it is possible to correlate the effective parameters and hyperbolic tangent functions constants.

The effective parameters to demonstrate partially saturated soils behaviour are stated as follows:

a. Parameters which describe a measure of soil degree of saturation, like Sr and moisture content (w).
b. Since plasticity indices have a great influence on fine-grained soils behaviour, thus, these parameters should be taken into account. LL, PL and PI can reflect the influence of soil plasticity characteristics.
c. Parameters that express the behaviour of soils in saturated state based on Mohr-Coulomb criterion. Internal friction angle in fully saturated and effective state $(\phi')$ and thus, M parameter in Mohr-Coulomb criterion can ensure the coincidence of the model in saturated situation.
d. Similar to fully saturated condition, in dry and hardened state, Hoek-Brown criterion was employed. Model parameters in dry condition can be expressed in Hoek-Brown parameters. Uniaxial compressive strength in dry state and close to altered rocks ($q_u$) and m parameter can satisfy the statement expressed here.
e. Consideration of soil suction in partially saturated soils.

Regarding the dependency of some previously mentioned parameters, selected effective parameters are moisture content (w), plasticity index (PI), M in Mohr-Coulomb criterion, m in Hoek-Brown criterion and uniaxial compressive strength (unconfined) in fully dry condition (qu).

Moisture content (w) can be simply determined in any soil mechanics laboratory. Thus, it can be a good measure for soil degree of saturation. Plasticity index (PI) is also a conventional parameter to describe soil properties that is usually determined in soil classification tests on fine-grained soils.

Parameter M in Mohr-Coulomb criterion can be attained by drained triaxial tests in saturated conditions.

Parameter m in Hoek-Brown criterion can be determined by the means of triaxial tests on intact rock and transforming the intact parameters into rock mass parameters by the means of GSI classification of rocks (Geological Strength Index) (Hoek and Brown, 2000).

There are also experimental values presented by the authors that can provide a range of m parameters based on rock type. Since rock like materials, i.e. fine-grained soils which are transformed into rock under heavy pressures, they are almost similar (clay-stone are the common type of these rocks).

Thus, m parameter varies in a closed range. In consequence, considering these parameters in a bracketed interval (lower limit to upper limit) can provide in a satisfactory results. Parameter m is suggested to be about 3.4 for clay stones. In this research, a range of m between 3.2 and 3.5 for stiff clays was determined as a good estimate. Parameter m can also be determined of uniaxial tests ($q_u$) for some rock types.

As stated before, soil suction is among the most significant parameters in partially saturated soils behaviour. This is why most of partially saturated soils behaviour models are functions of soil suction. Determination of soil suction is very hard during a test procedure in the laboratory. On the other hand, the instruments for testing partially saturated soils (e.g., partially saturated triaxial test apparatus) are not commonly used in traditional soil mechanics laboratories. Since the effect of soil suction cannot be ignored in a valid model of partially saturated soil, the effect of soil suction was replaced by other pertinent factors for practical purposes. The range of soil suction was also limited in (p,q,s) space to obtain more precise results.

Determination of a, b and c parameters in a hyperbolic function via w, PI, M, m and $q_u$ parameters was performed in a linear part by part curve-fitting procedure in Minitab software that is among the most conventional programs in step-by-step linear regression of multi-variable functions [17].

Based on the regression analyses, the most suitable parameters are determined as follows:

$$a = -4880 + 26.69\,w - 30.2\,PI + 1299\,m_{(HB)} + 709\,M + 0.215\,q_u \quad (5)$$

$$b = 0.0148 - 0.000119\,w + 0.000182\,PI - 0.00293\,m_{(HB)} - 0.00209\,M \quad (6)$$

$$c = 632 - 2.17\,w - 1.70\,PI - 144\,m_{(HB)} - 30.5\,M - 0.0463\,q_u \quad (7)$$

Correlation factor ($R^2$) for this set of equations is 96.8, 85.6 and 90.9 respectively. These values show a good correlation for input and output parameters.
The model was then validated by the experimental results for partially saturated fine-grained soils. According to collected data base, the constants of hyperbolic tangent function can be determined from equations 5 to 7 as follows:

$$a = 622 \quad , \quad b = 0.0018 \quad , \quad c = 15.8$$

Consequently hyperbolic tangent function can be presented as following:

$$q = 622\tanh(0.0018(p + 15.8)) \quad (8)$$

This function is shown in figure 11.

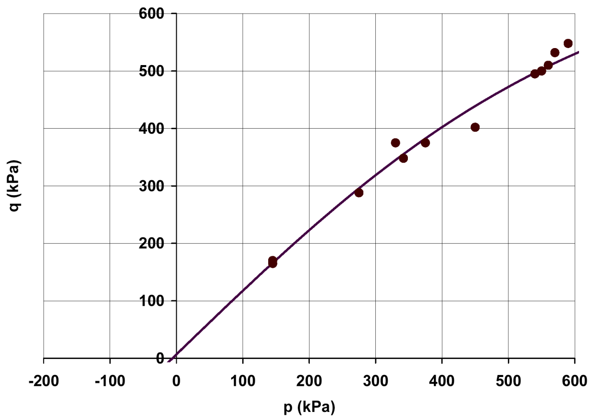


**Fig. 11.** Hyperbolic tangent function obtained from experimental results

Finally, estimated values of p and q, are compared with measured values. The comparison is presented in table 2 and shown in figure 12.
According to this comparison the uppermost error value in soil behaviour prediction is limited to 10% and less. Considering data scattering, the maximum error is reasonably acceptable. Thus, the model is applicable for partly saturated soils based on simple soil parameters.

**Table 2.** Comparison of measured and estimated values of deviator stress, q, based on hyperbolic tangent function

| No. | p | q (Measured) | q (Estimated) | Error (%) |
|---|---|---|---|---|
| 1 | 145 | 170 | 177 | 3.97 |
| 2 | 145 | 165 | 177 | 6.80 |
| 3 | 275 | 288 | 302 | 4.50 |
| 4 | 342 | 348 | 356 | 2.28 |
| 5 | 450 | 402 | 429 | 6.33 |
| 6 | 375 | 375 | 380 | 1.41 |
| 7 | 330 | 375 | 347 | 8.11 |
| 8 | 540 | 495 | 477 | 3.82 |
| 9 | 550 | 500 | 481 | 3.86 |
| 10 | 560 | 510 | 486 | 4.96 |
| 11 | 570 | 532 | 490 | 8.52 |
| 12 | 590 | 548 | 499 | 9.91 |

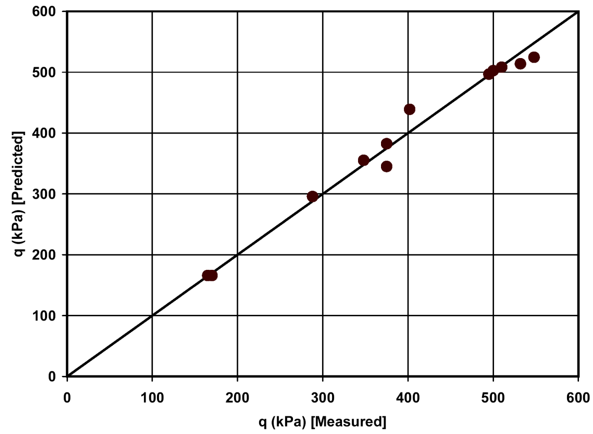


**Fig. 12.** Comparison of measured and estimated values of deviator stress, q, based on hyperbolic tangent function

## 4 Conclusion

In this research, a new behaviour model is presented based on collected experimental data of different researchers to express fine grained clayey soils behaviour in partially saturated state. The model was then validated with other experimental results. A transition of strength from fully saturated to fully dry conditions is the basis of the modelling. Model parameters are moisture content, plasticity indices, internal friction angle of saturated-drained condition and unconfined compressive strength in dry and hardened states.

Based on this equation and represented effective parameters, indirect estimate of shear strength of partially saturated soils is possible. The equation is a hyperbolic tangent function with a suitable correlation factor for training data and experimental results. In consequence, the model is applicable for practical purposes in geotechnical engineering, especially where advanced laboratory instruments are not in hand.

*References:*

[1] Ahmadi, H., "Evaluation of constitutive models for partially saturated soils; Application in road embankments", M.Sc. thesis, The University of Guilan, Iran, 2007.

[2] Borja, R. I., "Cam-Clay plasticity. Part V: A mathematical framework for three-phase deformation and strain localization analyses of partially saturated porous media", *Comput. Methods Appl*. Mech. Engrg., vol.193, 5301–5338, 2004.

[3] Fredlund, D.G., Xing, A., Fredlund, M.D., and Barbour, S.L., "The relationship of the unsaturated soil shear strength functions to the soil-water characteristic curve", *Canadian Geotechnical Journal*, Vol. 32, pp. 440-448, 1995.

[4] Sun, D., Sheng, D. and Sloan, S.W., "Elastoplastic modelling of hydraulic and stress–strain behaviour of unsaturated soils", *Mechanics of Materials*, In Press, 2006.

[5] Alonso, E. E., Gens, A. and Josa, A., "A constitutive model for partially saturated soils", *Géotechnique*, Vol. 40, No. 3, pp. 405-430, 1990.

[6] Josa A., Balmaceda, A., Gens A. and Alonso E. E., "An elastoplastic model for partially saturated soils exhibiting a maximum of collapse. Proc. 3rd Int. Computational Plasticity, Barcelona, 1992.

[7] Wheeler, S. J. and Sivakumar, V., "An elasto-plastic critical state framework for unsaturated soil", Géotechnique, Vol. 45, No. 1, pp.35-53, 1995.

[8] Bolzon, G., Schrefer, B. A. and Zienkiewicz, O. C., "Elastoplastic soil constitutive laws generalized to partially saturated", Géotechnique, Vol. 46, No. 2, pp. 279-289, 1996.

[9] Thomas, H. R. and He, Y., "Modelling the behaviour of unsaturated soil using an elastoplastic constitutive model", *Géotechnique*, Vol. 48, No. 5, pp. 589-603, 1998.

[10] Blatz, J.A., Graham, J. and Chandler, N.A., "Influence of suction on the strength and stiffness of compacted sand–bentonite", *Canadian Geotechnical Journal*, Vol. 39, pp. 1005–1015, 2002.

[11] Adams, B. A. and Wulfsohn, D., "Critical state behaviour of an agricultural soil", *Journal of Agricultural Engineering Research*, Vol. 70, pp. 345-354, 1998.

[12] Eko, R.M., "Use the triaxial stress state frameworke to evaluate the mechanical behaviour of an agricultural clay soil", *Soil & Tillage Research*, Vol. 81, pp. 71-85, 2005.

[13] Cetin, H., "Water content changes along shear planes in drained and undrained triaxial compression tests on unsaturated cohesive soils", *Tr. J.of Engineering and Environmental Science*, pp. 465-471, 1999.

[14] Rahardjo, H., Heng, O. B. and Choon, L. E., "Shear strength of a compacted residual soil from consolidated drained and constant water content triaxial tests", *Canadian Geotechnical Journal*, Vol. 41, pp. 421–436, 2004.

[15] Montanez, J. E. C., "*Suction and volume changes of compacted sand bentonite mixtures*", PhD thesis, University of London, Imperial College, 2002.

[16] Vanoudheusden, E., Sultan, N. and Cochonat, P., "Mechanical behaviour of unsaturated marine sediments: experimental and theoretical approaches", *Marine Geology*, Vol. 213, pp. 323-342, 2004.

[17] Minitab help, *http://www.minitab.com*, Minitab Inc., 1996.